Single and double π^-/π^+ ratios in heavy-ion reactions as probes of the high-density behavior of the nuclear symmetry energy

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Abstract

Based on an isospin- and momentum-dependent hadronic transport model IBUU04, effects of the nuclear symmetry energy on the single and double π^-/π^+ ratios in central reactions of ^{132}Sn+^{124}Sn and ^{112}Sn+^{112}Sn at a beam energy of 400 MeV/nucleon are studied. It is found that around the Coulomb peak of the single π^-/π^+ ratio the double π^-/π^+ ratio taken from the two isotopic reactions retains about the same sensitivity to the density dependence of nuclear symmetry energy. Because the double π^-/π^+ ratio can reduce significantly the systematic errors, it is thus a more effective probe for the high-density behavior of the nuclear symmetry energy.

PACS numbers: 25.70.-z, 25.60.-t, 25.80.Ls, 24.10.Lx

I. INTRODUCTION

After about three decades of intensive efforts in both nuclear experiments and theories, the equation of state (EOS) of isospin symmetric nuclear matter is now relatively well determined mainly by studying collective flows in heavy-ion collisions [1] and nuclear giant monopole resonances [2]. The major remaining uncertainty about the EOS of symmetric nuclear matter is due to our poor knowledge about the density dependence of the nuclear symmetry energy [1, 3, 4]. Therefore, the new challenge is to determine the EOS of isospin asymmetric nuclear matter, especially the density dependence of the nuclear symmetry energy. Besides the great interests in nuclear physics, the EOS of asymmetric nuclear matter is also crucial in many astrophysical processes, especially in connection with the structure of neutron stars and the dynamical evolution of proto-neutron stars [5]. Fortunately, heavy-ion reactions, especially those induced by radioactive beams, provide a unique opportunity to constrain the EOS of asymmetric nuclear matter [6, 7, 8]. In fact, considerable progress has been made recently in determining the density dependence of the nuclear symmetry energy around the normal nuclear matter density from studying the isospin diffusion in heavy-ion reactions at intermediate energies [9, 10, 11]. However, much more work is still needed to probe the high-density behavior of the nuclear symmetry energy.

A crucial task is to find experimental observables that are sensitive to the density dependence of the nuclear symmetry energy. A number of such observables have been already identified in heavy-ion collisions induced by neutron-rich nuclei, such as the free neutron/proton ratio [12], the isospin fractionation [13, 14, 15, 16, 17, 18], the neutron-proton transverse differential flow [19, 20, 21], the neutron-proton correlation function [22], $t/^3$He [23, 24], the isospin diffusion [25, 26], the proton differential elliptic flow [27] and the π^-/π^+ ratio [28, 29, 30, 31]. Generally, the long range Coulomb force on charged particles plays an important role in the above observables. It is thus important to distinguish the effects due to the symmetry potentials from those due to the Coulomb potentials. It is also useful to understand the interplay between these two kinds of potentials. Moreover, to extract accurately information about the symmetry energy one has to reduce as much as possible the systematic errors involved in the observables used in experiments. For this purpose one normally studies the ratios or relative values of experimental observables from two reaction systems using different isotopes of the same element. The first theoretical investigation of

such an observable, the double neutron/proton ratio of pre-equilibrium nucleons, has been made recently in Ref. [32]. Since the single π^-/π^+ ratio in heavy-ion collisions induced by neutron-rich nuclei has been shown to be a useful probe of the high-density behavior of the nuclear symmetry energy [28, 30, 31], we study here the effects of the symmetry energy on the double π^-/π^+ ratio from the reactions of ^{132}Sn+^{124}Sn and ^{112}Sn+^{112}Sn, i.e., the ratio of π^-/π^+ from ^{132}Sn+^{124}Sn over that from ^{112}Sn+^{112}Sn, using the IBUU04 model. It is well known that the single π^-/π^+ ratio has a Coulomb peak at certain pion kinetic energy depending on the system and the impact parameter of the reaction [33, 34, 35, 36]. It is thus especially interesting to examine the sensitivity of the double π^-/π^+ ratio to the symmetry energy around the Coulomb peak. We find that around the Coulomb peak the double π^-/π^+ ratio has about the same sensitivity to the symmetry energy as the single π^-/π^+ ratio while having the advantage of reduced systematic errors.

II. A BRIEF INTRODUCTION TO THE IBUU04 TRANSPORT MODEL

In the IBUU04 model, besides nucleons, Δ and N^* resonances as well as pions and their isospin-dependent dynamics are included. The initial neutron and proton density distributions of the projectile and target are obtained by using the relativistic mean field theory. The experimental free-space nucleon-nucleon (NN) scattering cross sections and the in-medium NN cross sections can be used optionally. In the present work, we use the isospin-dependent in-medium NN elastic cross sections from the scaling model according to nucleon effective masses [11]. For the inelastic cross sections we use the experimental data from free space NN collisions since the in-medium inelastic NN cross sections are still very much controversial. The total and differential cross sections for all other particles are taken either from experimental data or obtained by using the detailed balance formula. The isospin dependent phase-space distribution functions of the particles involved are solved by using the test-particle method numerically. The isospin-dependence of Pauli blockings for fermions is also considered. More details can be found in Refs. [11, 12, 37, 38, 39]. The momentum-dependent single nucleon potential (MDI) adopted here is [38]

$$
\begin{aligned}
U(\rho, \delta, \mathbf{p}, \tau) \; = \; & A_u(x)\frac{\rho_{\tau'}}{\rho_0} + A_l(x)\frac{\rho_\tau}{\rho_0} \\
& + B(\frac{\rho}{\rho_0})^\sigma(1 - x\delta^2) - 8x\tau\frac{B}{\sigma+1}\frac{\rho^{\sigma-1}}{\rho_0^\sigma}\delta\rho_{\tau'}
\end{aligned}
$$

$$+\frac{2C_{\tau,\tau}}{\rho_0}\int d^3\mathbf{p}'\frac{f_\tau(\mathbf{r},\mathbf{p}')}{1+(\mathbf{p}-\mathbf{p}')^2/\Lambda^2}$$
$$+\frac{2C_{\tau,\tau'}}{\rho_0}\int d^3\mathbf{p}'\frac{f_{\tau'}(\mathbf{r},\mathbf{p}')}{1+(\mathbf{p}-\mathbf{p}')^2/\Lambda^2}. \quad (1)$$

In the above equation, $\delta=(\rho_n-\rho_p)/(\rho_n+\rho_p)$ is the isospin asymmetry parameter, $\rho=\rho_n+\rho_p$ is the baryon density and ρ_n,ρ_p are the neutron and proton densities, respectively. $\tau=1/2(-1/2)$ for neutron (proton) and $\tau\neq\tau'$, $\sigma=4/3$, $f_\tau(\mathbf{r},\mathbf{p})$ is the phase-space distribution function at coordinate $\mathbf{r}$ and momentum $\mathbf{p}$. The parameters $A_u(x), A_l(x), B, C_{\tau,\tau},\ C_{\tau,\tau'}$ and Λ were set by reproducing the momentum-dependent potential $U(\rho,\delta,\mathbf{p},\tau)$ predicted by the Gogny Hartree-Fock and/or the Brueckner-Hartree-Fock calculations, the saturation properties of symmetric nuclear matter and the symmetry energy of about 32 MeV at normal nuclear matter density $\rho_0=0.16$ fm^{-3}. The incompressibility of symmetric nuclear matter at normal density is set to be 211 MeV. The parameters $A_u(x)$ and $A_l(x)$ depend on the x parameter according to

$$A_u(x)=-95.98-\frac{2B}{\sigma+1}x,\ \ A_l(x)=-120.57+\frac{2B}{\sigma+1}x, \quad (2)$$

where $B=106.35$ MeV. $\Lambda=p^0_F$ is the nucleon Fermi momentum in symmetric nuclear matter, $C_{\tau,\tau'}=-103.4$ MeV and $C_{\tau,\tau}=-11.7$ MeV. The $C_{\tau,\tau'}$ and $C_{\tau,\tau}$ terms are the momentum-dependent interactions of a nucleon with unlike and like nucleons in the surrounding nuclear matter. The parameter x is introduced to mimic various density-dependent symmetry energies $E_{\rm sym}(\rho)$ predicted by microscopic and phenomenological many-body approaches [40]. The isoscalar part $(U_n+U_p)/2$ [37, 39] of the single nucleon potential was shown to be in good agreement with that of the variational many-body calculations [41] and the results of the BHF approach including three-body forces [42]. The isovector part $(U_n-U_p)/2\delta$ [37] is consistent with the experimental Lane potential [43].

According to essentially all microscopic model calculations, see e.g., [44, 45], the EOS for isospin asymmetric nuclear matter can be expressed as

$$E(\rho,\delta)=E(\rho,0)+E_{\rm sym}(\rho)\delta^2+\mathcal{O}(\delta^4), \quad (3)$$

where $E(\rho,0)$ is the energy per nucleon of symmetric nuclear matter, and $E_{\rm sym}(\rho)$ is the nuclear symmetry energy. With the single particle potential $U(\rho,\delta,\mathbf{p},\tau)$, for a given value x, one can readily calculate the symmetry energy $E_{\rm sym}(\rho)$ as a function of density. Noticing that the isospin diffusion data from NSCL/MSU have constrained the value of x to be

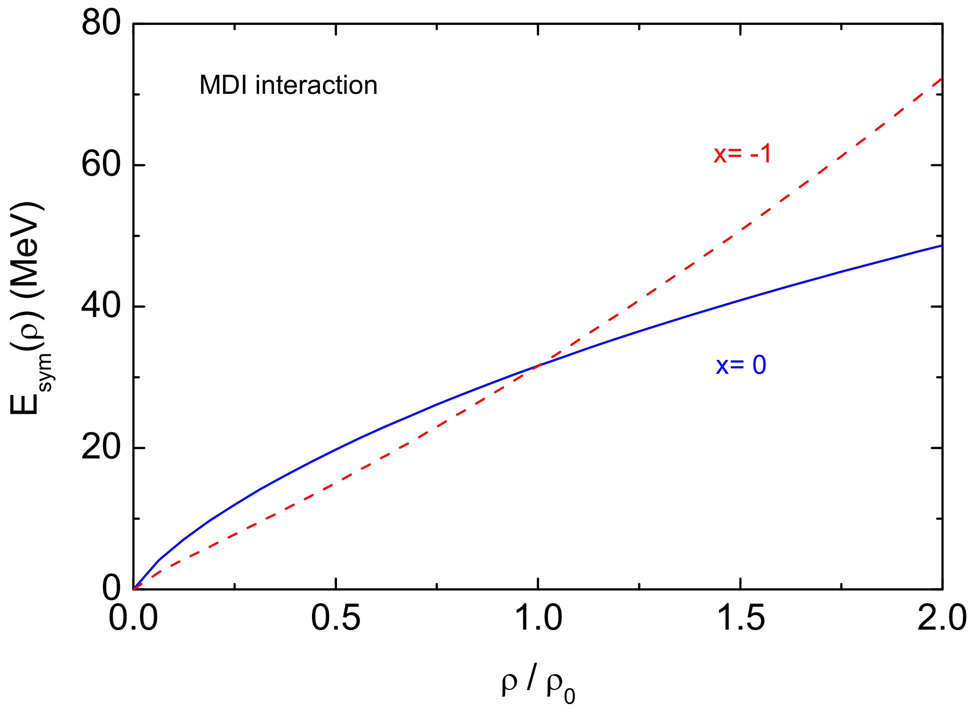

FIG. 1: (Color online) Density dependence of nuclear symmetry energy using the MDI interaction with $x = 0$ and $x = -1$. Taken from [32].

between 0 and -1 for nuclear matter densities less than about $1.2\rho_0$ [10, 11], in the present work we thus consider only the two values of $x = 0$ and $x = -1$. Shown in Fig. 1 is the density dependence of the nuclear symmetry energy with the two x parameters. It is seen that the case of $x = 0$ gives a softer symmetry energy than that of $x = -1$ and the difference becomes larger at higher densities.

III. RESULTS AND DISCUSSIONS

It was found earlier that the single π^-/π^+ ratio from heavy-ion collisions induced by neutron-rich nuclei can be used to probe the high density behavior of the nuclear symmetry energy [28, 29, 30, 31]. In Fig. 2, we show the kinetic energy distribution of the single π^-/π^+ ratio for the reactions of ^{132}Sn+^{124}Sn and ^{112}Sn+^{112}Sn at a beam energy of 400 MeV/nucleon and an impact parameter of $b = 1$ fm with the stiff ($x = -1$) and soft ($x = 0$) symmetry energy, respectively. In order to obtain good statistics, we used 12000

events for each reaction in the present work. It is seen that the overall magnitude of π^-/π^+ ratio is larger for the neutron-rich system ^{132}Sn+^{124}Sn than for the neutron-deficient system ^{112}Sn+^{112}Sn as expected. For the reaction ^{112}Sn+^{112}Sn the single π^-/π^+ ratio is not so sensitive to the symmetry energy due to the small isospin asymmetry. However, for the neutron-rich system ^{132}Sn+^{124}Sn the single π^-/π^+ ratio is sensitive to the symmetry energy and this is consistent with previous studies [28, 29, 30, 31]. It is further seen that the soft symmetry energy ($x = 0$) leads to a larger single π^-/π^+ ratio than the stiff one ($x = -1$). This is mainly because the high density region where most pions are produced are more neutron-rich with the softer symmetry energy as a result of the isospin fractionation [28].

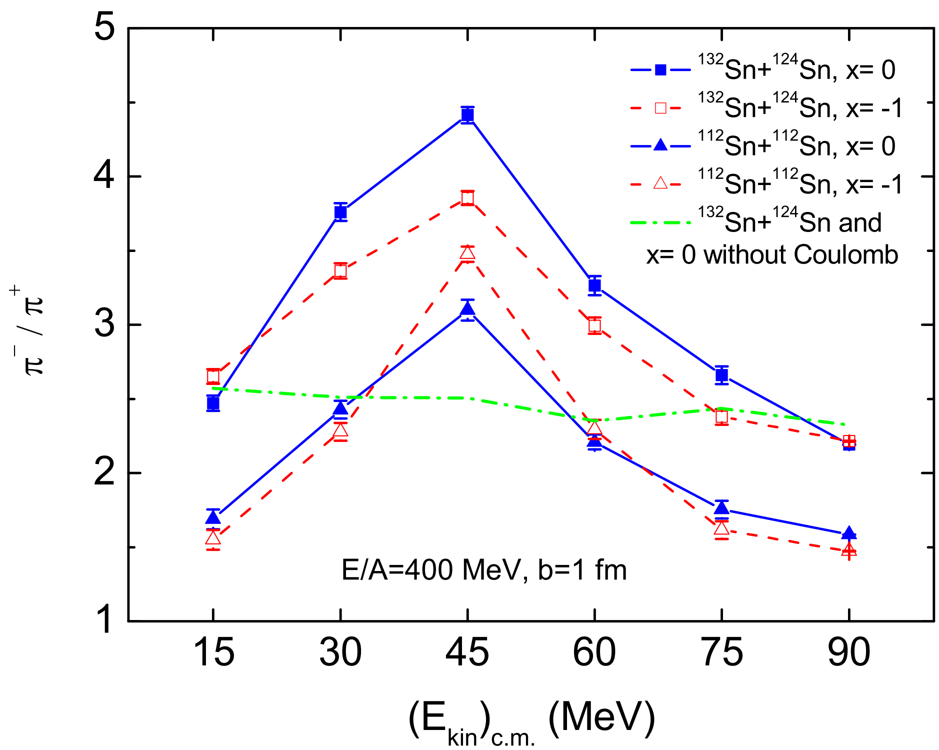

FIG. 2: (Color online) Kinetic energy distribution of the single π^-/π^+ ratio for ^{132}Sn+^{124}Sn and ^{112}Sn+^{112}Sn at a beam energy of 400 MeV/nucleon and an impact parameter of $b = 1$ fm with the stiff ($x = -1$) and soft ($x = 0$) symmetry energies. The dash-dotted line is the single π^-/π^+ ratio obtained by turning off the Coulomb potentials in the ^{132}Sn+^{124}Sn reaction.

From Fig. 2, it is also interesting to see that the single π^-/π^+ ratio exhibits a peak at a pion kinetic energy of about 45 MeV in all cases considered here. In order to understand the

origin of this peak, we also calculated the single π^-/π^+ ratios in both reactions by turning off the Coulomb potentials for all charged particles. As an example, shown in Fig. 2 with the dash-dotted line is the single π^-/π^+ ratio obtained by turning off the Coulomb potentials in the ^{132}Sn+^{124}Sn reaction. It is seen that the single π^-/π^+ ratio now becomes approximately a constant of about 2.4. The latter is what one expects based on the Δ resonance model [46]. According to the latter the π^-/π^+ ratio is approximately $(5N^2+NZ)/(5Z^2+NZ) \approx (N/Z)^2$ in central heavy-ion reactions with N and Z being the total neutron and proton numbers in the participant region [46]. For central ^{132}Sn+^{124}Sn reactions, the value is about 2.43. At 400 MeV/nucleon, pions are almost exclusively produced via the Δ resonances [47], it thus should not be a surprise to see the agreement with the Δ resonance model expectation. The comparison of calculations with and without the Coulomb potentials indicates clearly that the peak observed in the single π^-/π^+ ratio is indeed due to the Coulomb effects. The π^-/π^+ ratio carries some information about the symmetry energy mainly because it is sensitive to the isospin asymmetry of the nucleonic matter where pions are produced. This information might be distorted but is not completely lost because of the Coulomb interactions of pions with other particles. It is thus natural to look for signals of the symmetry energy in kinematic regions where the π^-/π^+ ratio reaches its maximum. In this regard, the Coulomb peak is actually very useful for studying the symmetry energy. It is necessary to stress that in many situations the Coulomb peak will simply appear at zero instead of a finite kinetic energy. One would then need to concentrate on the π^-/π^+ ratio of low energy pions. In addition, it should be mentioned that most of pions are produced in the high density nucleonic matter (about $2\rho_0$) through the Δ resonances and thus carry important information about the high density behavior of the symmetry energy. Since the pions at lower kinetic energies around the Coulomb peak experience many rescatterings with nucleons at high or low densities and the charged pions considered here also feel the Coulomb potential from protons at different densities, the information on high density symmetry energy from the lower energy pions may be distorted partially by the low density behavior of the symmetry energy [30]. However, in the present work, the soft ($x = 0$) and stiff ($x = -1$) symmetry energies have slight difference at low densities and the large difference appears at high densities (about $2\rho_0$) as shown in Fig. 1 and we thus expect the observed symmetry energy effects on the energy dependence of the π^-/π^+ ratio mainly reflect (though not completely) information on the high density behavior of the symmetry energy.

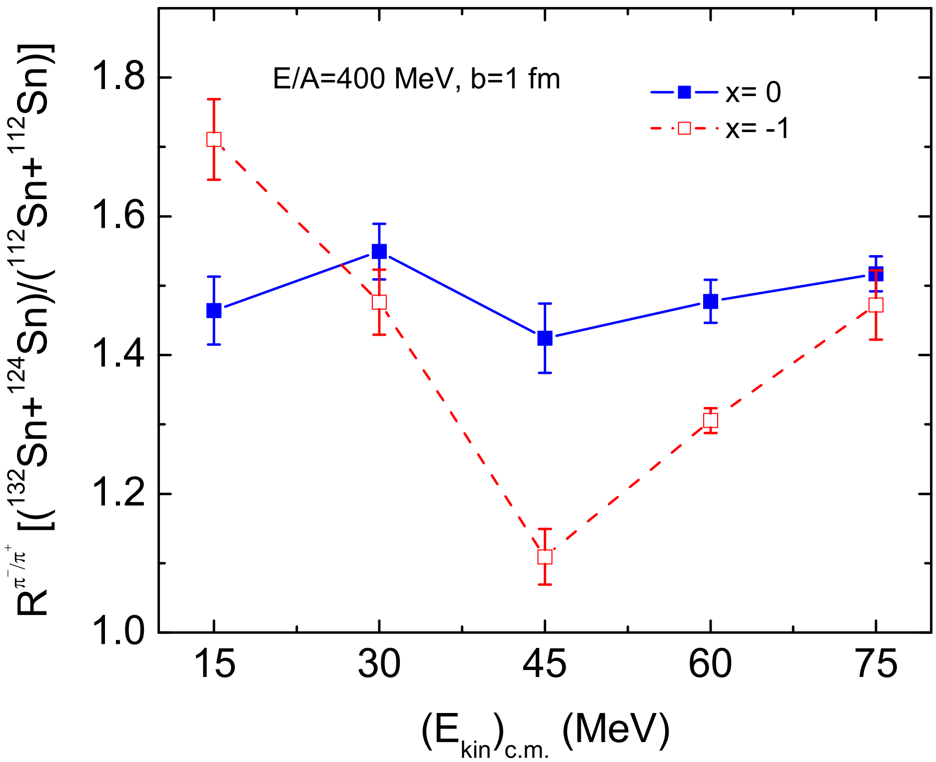

FIG. 3: (Color online) Kinetic energy dependence of the double π^-/π^+ ratio of ^{132}Sn+^{124}Sn over ^{112}Sn+^{112}Sn at a beam energy of 400 MeV/nucleon and an impact parameter $b = 1$ fm with the stiff ($x = -1$) and soft ($x = 0$) symmetry energies.

To reduce the systematic errors, it is more useful to study the double π^-/π^+ ratio in the reactions of ^{132}Sn+^{124}Sn and ^{112}Sn+^{112}Sn. A practically critical question is whether the sensitivity to the symmetry energy observed in the single π^-/π^+ ratio can be sustained by the double ratio. To answer this question we examine in Fig. 3 the double π^-/π^+ ratio for the two reactions considered. It is seen that the kinetic energy dependence of the double π^-/π^+ ratio is rather different for the stiff ($x = -1$) and soft ($x = 0$) symmetry energies. The double π^-/π^+ ratio is quite flat for $x = 0$ while displaying a concave structure for $x = -1$ around the Coulomb peak. These different behaviors can be understood from the corresponding single π^-/π^+ ratios in the two reactions as shown in Fig. 2. It is reassuring to see that around the Coulomb peak the double π^-/π^+ ratio is still sensitive to the symmetry energy. Compared with the single π^-/π^+ ratio, the kinetic energy dependence of the double π^-/π^+ ratio becomes weaker. This is because the effects of the Coulomb potentials are

reduced in the double π^-/π^+ ratio. We note that the double π^-/π^+ ratio displays an opposite symmetry energy dependence compared with the double n/p ratio for free nucleons shown in Ref. [32]. This is understandable since the soft symmetry energy leads to a neutron-richer dense matter in the heavy-ion collisions induced by neutron-rich nuclei and thus more π^-'s would be produced due to more neutron-neutron inelastic scatterings. Since the soft symmetry energy leads to a neutron-richer dense nucleonic matter, the n/p ratio for free nucleons is therefore expected to be smaller due to the charge conservation.

IV. SUMMARY

In summary, using the isospin- and momentum-dependent hadronic transport model IBUU04, we studied the single and double π^-/π^+ ratios and their dependence on the nuclear symmetry energy in the central reactions of ^{132}Sn+^{124}Sn and ^{112}Sn+^{112}Sn at a beam energy of 400 MeV/nucleon. We found that the double π^-/π^+ ratio retains the same sensitivity to the symmetry energy as shown by the single π^-/π^+ ratio around its Coulomb peak in the neutron-richer system involved. These features are useful for extracting information about the nuclear symmetry energy from future experimental data. Because the double π^-/π^+ ratio can reduce significantly the systematic errors, it is thus a more effective probe for the high-density behavior of the nuclear symmetry energy.

Acknowledgments

The work is supported by the US National Science Foundation under Grant No. PHY-0354572, PHY0456890 and the NASA-Arkansas Space Grants Consortium Award ASU15154, the National Natural Science Foundation of China under Grant No. 10105008, 10575071, 10575119, 10235030, the Chinese Academy of Science Knowledge Innovation Project (KJCX2-SW-N02), Major State Basic Research Development Program (G2000077400), and the Chinese Ministry of Science and Technology.

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